

A COMPREHENSIVE STUDY ON REAL-TIME MULTI-OBJECTIVE OPTIMIZATION FOR EFFICIENT WATER DISTRIBUTION IN CANAL SYSTEMS

ÉTUDE APPROFONDIE DE L'OPTIMISATION MULTIOBJECTIF EN TEMPS RÉEL POUR UNE DISTRIBUTION EFFICACE DE L'EAU DANS LES RÉSEAUX DE CANAUX

Mai Zijun¹, He Yupu¹, Ji Renjing¹, Zhang Wei¹, Qi Wei¹, and Zha Yuanyuan²

ABSTRACT

Traditional irrigation scheduling adopts macroscopic water balance, simplifying canals into pipes with fixed losses. These methods fail to capture dynamic water level changes at canal ends and unsteady flow recession, causing ineffective supply and tail-water loss. This paper proposes a method coupling macroscopic reservoir scheduling with microscopic hydrodynamic simulation. First, based on remote sensing and elevation data, the minimum effective water level for gravity flow at each main canal end is determined. An improved Manning formula back-calculates the minimum supply flow lower bound and pre-storage threshold, forming bottom-line constraints. Second, a mixed-integer linear programming (MILP) model minimizing total release, delivery duration, and gate operations is built to obtain reservoir release and canal diversion hydrographs. These serve as upstream boundaries of the 1D unsteady Saint-Venant equations solved by the Preissmann implicit scheme. The downstream boundary switches dynamically based on real-time comparison between canal-end depth and the effective water level. Net effective supply is integrated. The maximum deviation between net effective supply and target demand is the convergence criterion; otherwise, a damping learning rate corrects the macroscopic target, driving macro-micro iteration. A case study in Hubei, China, validates the method. Results show that under different demand scenarios, the method optimizes water distribution, meets gravity-flow requirements, reduces ineffective supply and tail-water loss, shortens delivery time, and decreases gate operations, solving the traditional 'volume satisfied but level insufficient' problem.

Keywords: Irrigation district optimal scheduling; multi-source water supply; effective gravity-flow water level; mixed-integer linear programming; macro–micro coupling.

RÉSUMÉ

Les méthodes classiques de programmation de l'irrigation reposent sur un bilan hydrique macroscopique et assimilent les canaux à des conduites présentant des pertes fixes. Elles ne rendent pas compte des variations dynamiques du niveau d'eau aux extrémités des canaux ni de la décrue en régime d'écoulement non permanent, ce qui entraîne des apports inefficaces et des pertes d'eau en aval. Cet article propose une méthode couplant la programmation macroscopique des réservoirs à une simulation hydrodynamique microscopique. Premièrement, à partir de données de télédétection et d'altitude, le niveau d'eau effectif minimal nécessaire à l'écoulement gravitaire à l'extrémité de chaque canal principal est déterminé. Une formule de Manning améliorée permet de calculer en retour la borne inférieure du débit minimal d'alimentation et le seuil de préstockage, qui constituent les contraintes minimales. Deuxièmement, un modèle de programmation linéaire en nombres entiers mixtes (MILP), visant à réduire au minimum le volume total lâché, la durée d'acheminement et le nombre de manœuvres des vannes, est construit afin d'obtenir les hydrogrammes de lâcher des réservoirs et de dérivation dans les canaux. Ceux-ci servent de conditions aux limites amont des équations unidimensionnelles (1D) de Saint-Venant en régime non permanent, résolues par le schéma implicite de Preissmann. La condition limite aval s'ajuste dynamiquement en fonction de la

1 Nanjing Hydraulic Research Institute, Rural Water Conservancy Research Institute, Nanjing, China, Email: zjmai@nhri.cn

2 Wuhan University, Wuhan, China Email: 437153280@qq.com

comparaison, en temps réel, entre la profondeur à l'extrémité du canal et le niveau d'eau effectif. L'apport effectif net est ensuite intégré. L'écart maximal entre l'apport effectif net et la demande cible sert de critère de convergence; à défaut, un taux d'apprentissage amorti corrige la cible macroscopique et pilote l'itération macro-micro. Une étude de cas menée dans la province du Hubei, en Chine, valide la méthode. Les résultats montrent que, selon différents scénarios de demande, cette méthode optimise la distribution de l'eau, satisfait les exigences d'écoulement gravitaire, réduit les apports inefficaces et les pertes d'eau en aval, raccourcit la durée d'acheminement et diminue le nombre de manœuvres des vannes, résolvant ainsi le problème classique du « volume satisfait, mais niveau insuffisant ».

Mots clés : Programmation optimale du périmètre irrigué; Approvisionnement en eau provenant de sources multiples; Niveau d'eau effectif gravitaire; Programmation linéaire à nombres entiers mixtes; Couplage macro-micro.

1. INTRODUCTION

Irrigated agriculture consumes about 70% of global freshwater. Improving irrigation efficiency is critical for water and food security. In large irrigation districts, water is conveyed from upstream reservoirs to farmlands via main canals and finally enters fields by gravity flow, which requires the canal-end water depth to exceed a threshold determined by field elevation and inlet geometry. Traditional scheduling methods adopt macroscopic water balance, treating canals as pipes with fixed losses, using total supply or deficit as objectives and cumulative flow as the evaluation basis (Ilich et al., 2024). These methods have three defects: they ignore dynamic water levels at canal ends, so water volume may be satisfied but actual depth is below the effective level, causing ineffective supply; they ignore unsteady recession, so tail water loses gravity capacity after gate closure; the macroscopic model cannot couple hydraulic response, making optimized hydrographs difficult to execute (Tekle et al., 2025; Nazari et al., 2024). Recent research trends toward refinement and coupling. MILP handles discrete decisions in canal scheduling. Joint scheduling of series- parallel reservoirs and canals has been studied (Wu et al., 2024).

The 1D Saint-Venant equations and Preissmann scheme are standard for unsteady flow simulation (Du et al., 2026), and water system cybernetics provides a systematic framework (Lei et al., 2025). Simulation-optimization coupling has been applied to irrigation canals, reducing flow fluctuations and improving delivery efficiency (Fan et al., 2023). However, existing coupling frameworks seldom embed the core physical condition-the effective gravity-flow water level at the canal end-into the optimization loop. Another persistent problem is tail-water loss. After gate closure, the water remaining in the canal continues flowing downstream but its depth drops below the effective threshold, becoming unusable-a phenomenon ignored in steady-state models. This necessitates embedding the canal-end effective water level as a physical bottom-line constraint (Zhang et al., 2025).

To address these issues, this paper proposes an optimal scheduling method based on the effective gravity-flow water level at canal ends. Contributions: (1) a physical bottom-line constraint that back-calculates minimum supply flow and pre-storage threshold from the minimum effective level; (2) the Gravity-Flow Effective Index (GFEI) to distinguish effective from ineffective supply; (3) a macro-micro closed-loop iterative framework coupling MILP and 1D unsteady flow, minimizing released volume, delivery duration, and gate operations. A case study in Hubei, China, validates the method.

2. METHODOLOGY

2.1 Macroscopic MILP Optimization Model

2.1.1 Objective Function

Taking reservoir release flow, canal head diversion flow, gate status as decision variables, an objective function containing three- dimensional penalty factors-minimize released water volume, minimize water delivery duration, and minimize gate operation duration – is constructed:

$$\text{Min}\{\sum_{i,t}(C_1 \cdot R_{i,t}) + \sum_{j,t}(C_2 \cdot t \cdot D_{j,t}) + \sum_{j,t}(C_3 \cdot Y_{j,t})\} \quad (1)$$

Where $R_{i,t}$ is the release flow of reservoir i at time (t) (m^3/s), $D_{j,t}$ is the diversion flow at the head of main canal (j) at time t (m^3/s), $Y_{j,t} \in \{0,1\}$ is the gate on/off state variable of main canal j at time (t) , and C_1, C_2, C_3 are weight coefficients. The first term minimizes the total released water volume of the system, saving water resources while ensuring water supply. The second term introduces a time penalty factor t (time step index, increasing linearly with the scheduling process), which together with the weight C_2 drives the optimizer to spontaneously allocate water rights to the early part of the schedule-the same diversion flow at a later time receives a larger penalty, encouraging the optimization scheme to complete the water delivery task as soon as possible and shorten the canal water delivery duration. The third term penalizes the gate opening state $Y_{j,t}$, with weight C_3 suppressing frequent gate operations and driving concentrated flow release to reduce the total gate opening time.

The branch-and-bound method is used to solve the above MILP model. , and the decision tensor is the input for each macro-micro iterative optimization.

2.2.2 Constraints

The model must satisfy the following multi-dimensional engineering constraints:

$$\text{Reservoir water balance constraint: } S_{i,t} = S_{i,t-1} + (I_{i,t} - R_{i,t} - QA_{i,t})\Delta t \quad (2)$$

$$\text{System supply-demand water volume constraint: } \sum_{i,t} R_{i,t} \Delta t \geq \sum_j^J \text{Demand}_j \quad (3)$$

$$\text{Canal water delivery satisfaction constraint: } D_{j,t} \Delta t \geq \text{Demand}_j \quad (4)$$

Where $S_{i,t}, I_{i,t}, R_{i,t}, QA_{i,t}$ are the storage (m^3), natural inflow (m^3/s), release flow (m^3/s), and spillage (m^3/s) of reservoir i at time t , respectively, Δt is the duration of each time step (s). Demand_j is the target water demand (m^3) of the irrigation plot corresponding to main canal j within the scheduling period, and J is the total number of canals. Other constraints include reservoir discharge flow, water level amplitude, reservoir water level, storage capacity constraints, canal flow, water level constraints, etc.

2.3 Bottom-Line Constraints

2.3.1 Minimum Effective Gravity-Flow Water Level

Based on remote sensing images and UAV images of the irrigation district and elevation data, and by mapping the elevation relationship between irrigation plots and the control cross-sections at the canal ends, combined with irrigation district operation management experience, the minimum effective gravity-flow water level $Z_{eff,j}$ required at the end of each canal system to achieve gravity flow is comprehensively determined. Specifically, reference can be made to the team's previous research achievements (Mai et al., 2024). This water level is the physical benchmark for judging whether the water delivered at the canal end constitutes effective water supply.

2.3.2 Lower Bound of Minimum Water Supply Flow

After determining the minimum effective gravity-flow water level $Z_{eff,j}$, it is necessary to back-calculate the corresponding inlet flow lower bound $Q_{min,j}$. The Manning formula is introduced for calculation:

$$Q_{min,j} = \frac{1}{n_j} \cdot (B_j \cdot Z_{eff,j}) \cdot \left(\frac{B_j \cdot Z_{eff,j}}{B_j + 2Z_{eff,j}} \right)^{2/3} S_{0,j}^{1/2} \quad (5)$$

Where n_j the Manning roughness coefficient of canal j is, B_j is the bottom width (m), and $S_{0,j}$ is the bottom slope.

2.3.3 Pre-storage Flow Threshold

To avoid scouring damage to the canal caused by a sudden increase in flow at the beginning of water supply and to achieve a smooth rise in water level, a small flow should be used to fill the initial canal storage before formal water supply. The pre-storage flow $Q_{pre,j}$ threshold is defined as a certain proportion of the lower bound of the minimum water supply flow $Q_{min,j}$: $Q_{pre,j} = \beta_j \cdot Q_{min,j}$. β_j is the pre-storage coefficient.

2.4 Gravity-Flow Discriminant Function

2.4.1 Gravity-Flow Effective Index (GFEI) at the Canal End

To quantitatively judge whether the water output at the canal end constitutes effective water supply, a binary gravity-flow discriminant index is defined:

$$GFEI_j(t) = \begin{cases} 1, & (Z_j(L, t) - Z_{eff,j}) \geq 0 \\ 0, & (Z_j(L, t) - Z_{eff,j}) < 0 \end{cases} \quad (6)$$

Where $Z_j(L, t)$ is the water depth (m) at the end (L) of main canal (j) at time (t), and $GFEI_j(t)$ is the gravity-flow discriminant index of main canal j at time t .

2.4.2 Initial Canal Storage Capacity and First-Iteration Macroscopic Target

In the model construction, the initial canal storage capacity determined by the geometric parameters of the main canals and the minimum effective gravity-flow water level is incorporated into the macroscopic water supply target of the first iteration:

$$\text{Target}^{(1)} = \sum_{j=1}^J \text{Demand } j + \sum j = 1^J (B_j \cdot L_j \cdot Z_{eff,j}) \quad (7)$$

Where L_j is the length of the main canal (m), and J is the total number of canals.

2.5 Microscopic Canal Hydraulic Simulation

2.5.1 Dynamic Boundary Condition Based on GFEI

The water flow in the canal is described by the Saint-Venant equations. The upstream boundary condition comes from the decision tensor output. The downstream boundary condition is dynamically switched according to the GFEI :

1. **Pre-storage period:** When the canal-end water depth $Z_j(L, t) < Z_{eff,j}$, it is judged as the pre-storage period, the downstream gate is closed, and the boundary condition is set as $Q(L, t) = 0$. The upstream incoming water is forced to be intercepted and converted into dynamic storage, gradually raising the water level in the canal.
2. **Water supply period:** When $Z_j(L, t) \geq Z_{eff,j}$, the GFEI index jumps to 1, triggering the gate to open, and the downstream boundary becomes free outflow, releasing effective water.

2.5.2 Net Effective Water Supply Integration

After obtaining the outflow at the canal end, the effective water volume is isolated and integrated through the GFEI index: $V_{eff,j}^{(k)} = \int_0^T Q_{out,j}(t) \cdot GFEI_j(t) \cdot dt$. $V_{eff,j}^{(k)}$ is the net effective water supply (m^3) of

main canal (j) in the k-th iteration, $Q_{out,j}(t)$ is the output flow (m^3/s) at the end of main canal j at time (t), T is the total duration of the scheduling period (s).

2.6 Error Feedback and Iterative Convergence

The water supply deviation of main canal (j) in the k-th iteration is: $Error_j^{(k)} = Demand_j - V_{eff,j}^{(k)}$.

Calculate the maximum absolute deviation $Error_{max}^{(k)}$ among all main canals, If $Error_{max}^{(k)}$ is greater than or equal to the preset engineering tolerance ε and the current iteration number k is less than the maximum iteration number K, the following damping error feedback formula is used to update the macroscopic water supply target for the next iteration: $Target^{(k+1)} = Target^{(k)} + \alpha \cdot Error_{max}^{(k)}$. α is the

system damping learning rate, range (0,1]. The updated $Target^{(k+1)}$ is used as the new right-hand side of the supply-demand balance constraint in the mixed-integer linear programming model, driving the iterative optimization until convergence or the maximum iteration number is reached, forming a macro-micro closed-loop iteration. When $Error_{max}^{(k)} \leq \varepsilon$, the optimal scheduling plan is output. If $k = K$ and convergence is still not achieved, the suboptimal approximation plan with the smallest absolute deviation among the historical iteration records is output.

3. CASE STUDY AND DATA

3.1 Study Area Overview

An irrigation district in Hubei Province, China, is taken as the research object. This irrigation district is one of the largest reservoir gravity irrigation districts in southern China, mainly planting staple crops such as rice, with a designed irrigation area of about 2.6 million mu. It has formed a water conservancy irrigation network with the Reservoir as the backbone, small and medium-sized water facilities as the foundation, and pumping stations as a supplement, combining large, medium, small, and micro scales, and integrating storage, diversion, lifting, and drainage. It is a large-scale water conservancy project mainly for agricultural irrigation, with comprehensive benefits including flood control, urban water supply, power generation water supply, aquaculture, shipping, tourism, and ecological water replenishment. The main canal head of the irrigation district is connected to Reservoir 1, and the middle part connects to other Reservoirs, forming a multi-source, multi-diversion engineering structure. In this paper, the reservoir-canal system of the irrigation district is partitioned and decoupled, forming a core operation unit—a system comprising five reservoirs (Reservoirs 1, 2, and 3 connected in series on the main stream, and Reservoirs 4 and 5 connected in parallel on tributaries) and four main canals. Its topological structure is shown in Figure 1. The maximum safe controlled flow limit of the main canal is $115 m^3/s$.

3.2 Basic Parameters

The basic parameters of the reservoirs and canals in the irrigation district are shown in Tables 1 and 2. Frequency analysis of the single-irrigation water demand of the irrigation plots controlled by each canal is carried out, and the water demands at 50%, 75%, and 90% frequencies (ten thousand m^3) are substituted into the model for calculation and solution.

Table 1. Basic parameters of canals in the irrigation district

index	B (m)	L (km)	n	S_0	Z_{eff} (m)	50%	75%	90%
1	25	83.3	0.022	0.0003	0.5	2160	2409	2640
2	20	90	0.022	0.0003	0.7	1800	2030	2200
3	15	56	0.022	0.0003	0.5	288	320	352
4	25	120	0.022	0.0003	0.3	6120	6810	7480

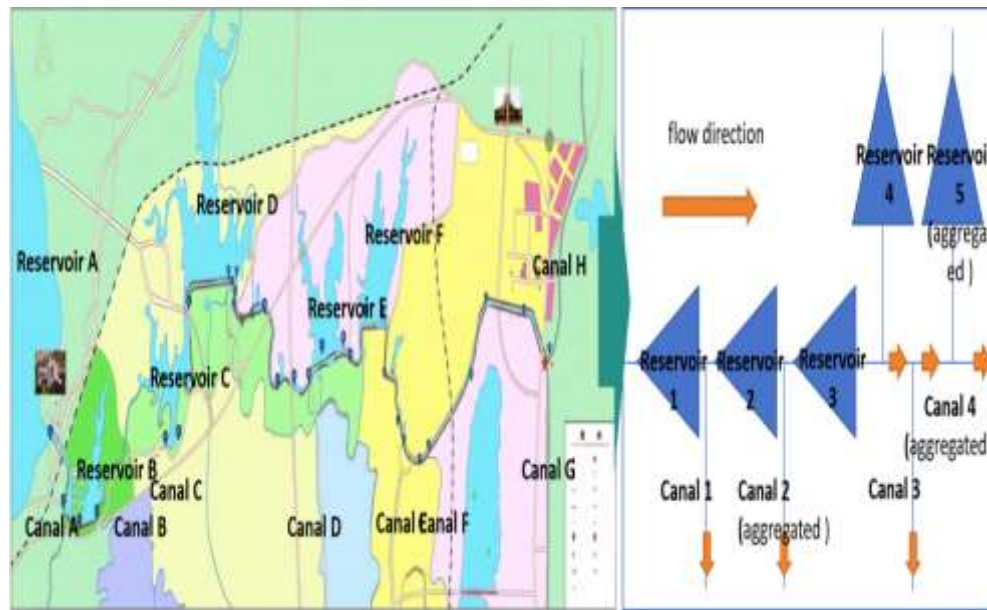


Figure 1. Topological diagram of the irrigation district reservoir and canal system

Table 2. Basic parameters of reservoirs in the irrigation district

index	total storage (ten thousand m ³)	utilizable storage	dead storage	the set water flow (m ³ /s)	initial storage
1	211300	92400	86200	5.8	50000
2	2122	717	61	0	500
3	615	454	18	0	300
4	970	688	45	0.6	450
5	520	189	611	0.6	100

3.3 Model Parameter Configuration

The scheduling period is set to 20–50 days according to the water demand magnitude. The time step Δt is set to days for the macroscopic model and seconds for the microscopic model. The MILP weights are determined by trial calculation, $C_1: 100, C_2: 5, C_3: 50$. The pre-storage coefficient β_j is set to 0.2. The damping learning rate α is set to 0.75. The convergence tolerance ε is set to 1% of the water demand. The maximum iteration number K is set to 20.

4 RESULTS

4.1 Macroscopic Optimization Results

The reservoir storage and release processes obtained by solving the MILP model are shown in Figure 2. When Reservoir 1 upstream needs to release a large amount of water, the flow constraint of the main canal $115\text{m}^3/\text{s}$ is automatically activated. The model triggers Reservoirs 2 and 3 to release storage in advance and share the release flow when the peak arrives, achieving peak shaving and staggering. At the same time, Reservoirs 4 and 5 provide lateral supply, ensuring that constraints such as flow ramp limits are satisfied. Reservoir releases are concentrated in the early part of the scheduling period (first 10 days), and the release gradually decreases in the later period. From the

analysis of the results under 50%, 70%, and 90% water demand conditions, as water demand increases, the reservoir release duration obviously increases. Considering that Reservoirs 2–5 have relatively small storage capacities compared with Reservoir 1 and limited regulation ability, under the three water demand conditions, the storage at the end of the scheduling period are all only dead storage.

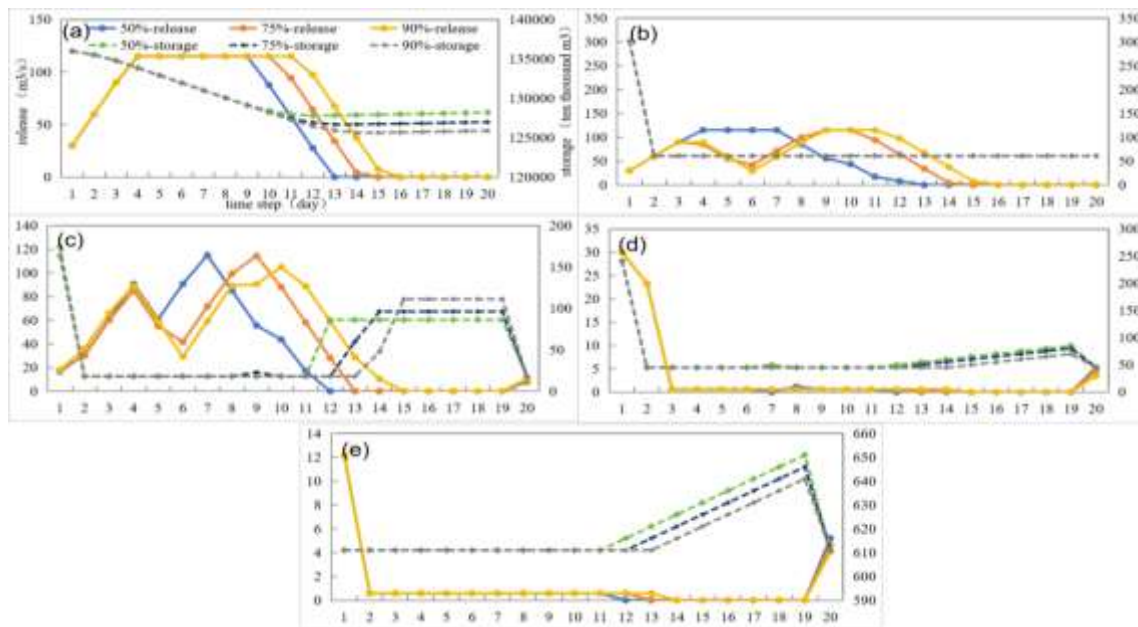


Figure 2. The reservoir capacity and discharge process of the reservoir group.(a) to (e) represent reservoirs 1 to 5 respectively.

4.2 Microscopic Hydraulic Simulation

The flow at the ends of each canal are shown in Figure 3, and the cumulative water volume processes are shown in Figure 4. The figures show that all four canals complete the water delivery requirement. Canal diversion flows are relatively concentrated in the early scheduling period, and under the 50%, 75%, and 90% water demand conditions, canal water delivery is completed on days 13, 14, and 15, respectively, showing significant timeliness. The object effectively drives the optimizer to allocate water rights to the early part of the schedule. In terms of gate operation, and significantly reduces the operation frequency. The gate opening duration of each canal is concentrated in 1–3 consecutive time steps, avoiding frequent switching and reducing operation cost and hydraulic disturbance. From the analysis of the results, the total inflow of the system is 12.0 million m³. Under the 50%, 75%, and 90% water demand conditions, the reservoir group consumes 93.60, 105.61, and 116.64 million m³ of storage, respectively. The effective water above the gravity-flow water level meets the water demand requirements. Excluding conveyance seepage and evaporation losses, the effective water reaching the canal end accounts for 98.5%, 98.6%, and 98.7% of the total water, respectively, with only a small amount of water remaining at the canal end as storage compensation.

4.3 Comparison with Traditional Methods

To verify the improvement effect of the proposed method, a traditional macroscopic water balance optimization was implemented for the same irrigation district. When the release processes were input into the Saint-Venant model for simulation verification, the actual water depths at the ends of the four canals frequently fell below the effective gravity-flow water level, and the effective water volume delivered to the ends was less than 80% of the target water demand; the number of gate operations increased by an average factor of four. The proposed method embeds the GFEI index into the optimization closed loop, enabling the macroscopic target to sense the microscopic hydraulic response, autonomously correcting the water distribution plan during the convergence process, and

finally achieving an accurate match between the effective water supply and the actual water demand within the tolerance. For irrigation districts already equipped with automatic hydrological telemetry systems and remote gate control, the proposed macro–micro coupling framework can be integrated with existing SCADA systems to achieve data exchange and automatic decision support through APIs. Due to space limitations, only the decoupled topology of the irrigation district is used for calculation; the restoration of the topology after optimization is not expanded here.

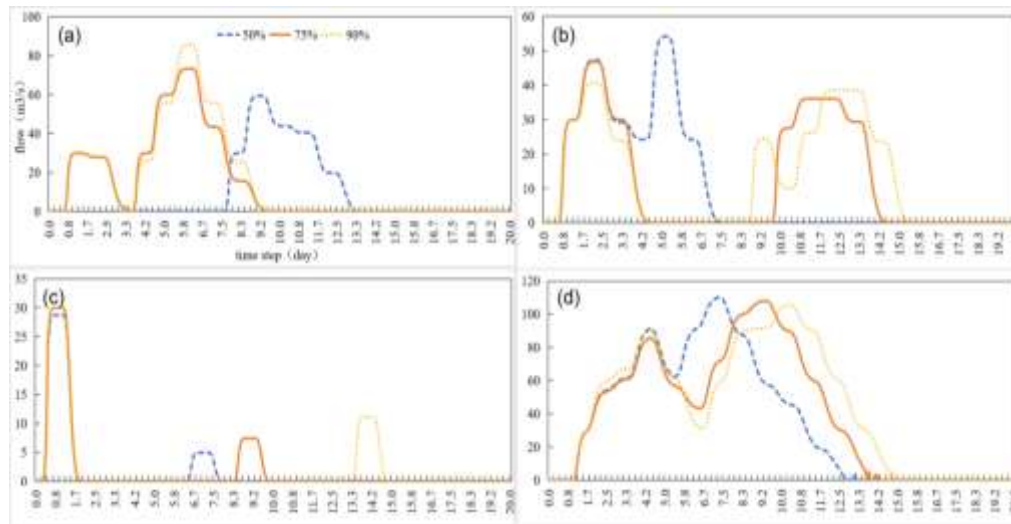


Figure 3. The flow process at the end of each canal under different water demand conditions. (a) to (d) represent canals 1 to 4 respectively.

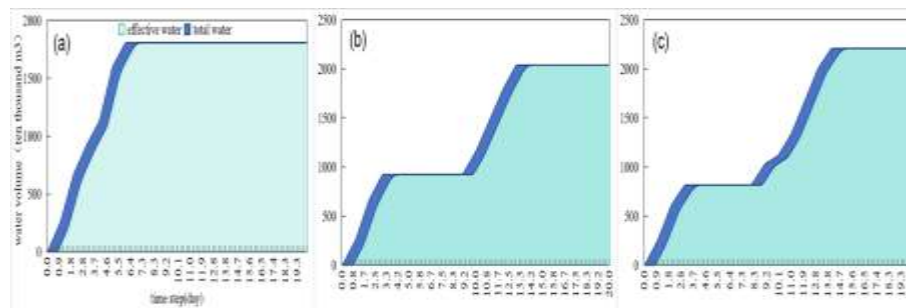


Figure 4. The cumulative water volume process of canal 2 under different water demand conditions. (a)50%,(b)75%,(c)90%.

5 CONCLUSION

This paper proposes an optimal scheduling method for multi-source reservoir-canal systems in irrigation districts based on the effective gravity-flow water level at canal ends. The main conclusions are as follows:

1. A physical bottom-line constraint method is established that back-calculates the lower bound of the minimum water supply flow and the pre-storage threshold from the minimum effective gravity-flow water level, converting the canal-end gravity-flow condition into an executable flow boundary in the scheduling model.
2. Through the combined design of the time penalty factor and the gate opening/closing penalty term in the MILP objective function, three-dimensional synergistic optimization of 'saving water resources- shortening water delivery period-reducing operation cost' is achieved.

3. Based on the Preissmann implicit difference scheme and the GFEL index dynamic boundary switching mechanism, a microscopic hydraulic simulation model that accurately depicts the whole process of canal pre-storage filling-reaching the target and opening the gate-effective water supply is constructed. The method of integrating the net effective water supply physically excludes non-gravity ineffective water during the pre-storage period and recession process.

REFERENCES

- Du Y., Wang W., Cao Q., et al. Simulation study on hydraulic transition process of branching channels under gate regulation[J]. Transactions of the Chinese Society of Agricultural Engineering, 2026, 42(01):132-141. (in Chinese)
- Fan Y., Chen H., Gao Z., et al. A model coupling water resource allocation and canal optimization for water distribution[J]. Water Resources Management, 2023, 37(3): 1341-1365.
- Ilich, N., & Todorović, A. Practical applicability of mathematical optimization for reservoir operation and river basin management: a state-of-the-art review. Hydrological Sciences Journal, 2024,69(13), 1881-1896.
- Lei X., Long Y., Xu H., et al. Cybernetics of hydro systems: Background, framework and research paradigm[J]. South-to-North Water Transfers and Water Science & Technology, 2025, 23(04):761-769+904.(in Chinese)
- Mai, Z., He, Y., Feng, C., et al. Multi-objective modeling and optimization of water distribution for canal system considering irrigation coverage in artesian irrigation district. Agricultural Water Management,2024, 301, 108959.
- Nazari, M., & Kerachian, R. Optimal operation of reservoirs considering water quantity and quality aspects: A systematic State-of-the-Art review. Water Resources Management, 2024,38(15), 5911-5944.
- Tekle, S. L., Bonaccorso, B., & Naim, M. Simulation-based Optimization of Water Resource Systems: A Review of Limitations and Challenges: SL Tekle et al. Water Resources Management, 2025,39(2), 579-602.
- Wu T., Guan G., Chen X., et al. Study on the Control Mode of Multi-stage Channel and Reservoir Group Combined WaterTransfer in Large Irrigation Area[J]. Water Saving Irrigation, 2024(11): 119-128. (in Chinese)
- Zhang C., Yuan Y., Dong J., et al. Canal System Water Resource Optimization Considering Water Allocation Equity[J]. Transactions of the Chinese Society for Agricultural Machinery, 2025,56(11):687-695. (in Chinese)